THE FLOW RATE AND THE HOLD-UP OF SOLIDS UNDER THE COUNTER-CURRENT FLOW OF LIQUID PHASE IN A VIBRATING PLATE COLUMN

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Flow rates an dmean hold-ups of solids have been measured under the counter-current flow of water in a 50 mm in diameter vibrating perforated plate column. Experimental data have been described by a mathematical model expressing the specific flow rate of solids in dependence on the hold-up, terminal velocity of the particles, porosity of the plate, specific flow rate of the continuous phase and the frequency and amplitude of plate vibrations. It has been found that for systems exhibiting low particle terminal velocity the pumping effect of the plates may increase the flow rate of the dispersed phase to a value corresponding to the flow in the empty column.

Vibrating plate columns have found broad application in counter-current contacting of liquids in extraction technologies particularly for their high efficiency at high flow rates of phases. Vibrations of perforated plates induce at low energy costs turbulence uniformly over the whole column cross section, necessary to disperse the liquid and to achieve intensive mass transfer under relatively low longitudinal mixing of the phases[1]. For these features and considerable versatility, effected by modifications of the plates, these columns appear prospective for counter-current contacting of solids with a liquid. Solids/liquid systems are primarily ion exchangers for ion separations or as catalysts. Vibrating motion of the plates in these cases enhances mass transfer between the solids and liquid, or, with a suitable plate design, may increase or regulate the residence time of the solids within the column (*e.g.* for continuous replacement of inactivated catalysts, *etc.*). Safe design of such solumns requires information about the hydrodynamics of the dispersion in the column. While the motion of the dispersed solids in an empty column has been studied in detail and relationship describing this motion based on the fluidized bed hydrodynamics measurement have been available, reliable analogous data regarding the hydrodynamics of the vibrating plate column are missing.

The aim of this work has been to obtain experimental information about the flow rate of solids through the column in dependence on the solids hold-up in the column, geometrical configuration of the plate and the frequency and amplitude of vibrations. Perforated plates performing vibrating motion in the column affect the passage of solid particles in a complex way. By constricting the area of cross section available for the flow of phases they contribute to the increased hold-up on the plate and bring about relative decrease of the flow rate of solids. Owing to the reciprocating motion of the plates, however, part of the dispersion is being pumped through the plate openings from the space above and below the plate. The result of this pumping effect is an extra flow of the solids from the higher hold-up into the lower hold-up region which positively affects the net flow of the solids. The action of these competing factors compounds to a complex dependence of the flow rate of the dispersed phase on the hold-up, plate free area, flow rate of liquid and intensity of plate vibrations.

THEORETICAL

In the derivation of the relations for the description of the flow of the solids through the vibrating plate column we shall confine ourselves to the simple monodisperse case. From the practical standpoint it appears useful to compare this flow rate with the flow rate of the solids under the same mean hold-up in an empty column without the plates. Particle density shall be assumed greater than that of liquid.

The flow rate of solids through an empty column. In order to obtain an expression for the specific flow rate of solids (*i.e.* the flow rate per unit area of column cross section) in dependence on the hold-up and the specific flow rate of liquid it is necessary to know the dependence of the relative velocity of particles with respect to liquid, u_r, on the terminal velocity of a particle in the quiescent liquid, u_0. This dependence may be found best from measurement of the fluidized bed expansion under variable liquid flow rate while the height of the bed yields the hold-up of particles in the fluidized bed with sufficient accuracy. Such data may be desciibed very precisely by the relation proposed by Richardson and Zaki[2]

$$U_c = u_0(1 - X)^\alpha \qquad (1)$$

while estimating the mean particle terminal velocity, u_0, and the exponent α as parameters. The exponent α depends somewhat on the properties of the system.

From the expression for the relative velocity of phases as

$$u_r = U_d/X + U_c/(1 - X) \qquad (2)$$

and Eq. (*1*) the following expression results for the flow rate of the dispersion through an empty column

$$U_d = u_0X(1 - X)^{\alpha-1} - XU_c/(1 - X) \qquad (3)$$

while for the particle velocity with respect to the column we obtain

$$u = u_0(1 - X)^{\alpha-1} - U_c/(1 - X)\,. \qquad (4)$$

From the literature it has been known[3-5] that the terminal velocity of a particle in turbulent media and pulsating liquid is less compared to that in quiescent liquid. As the motion of the plates in the column induces intensive turbulence of liquid, this effect must be also taken into consideration. On assuming that the factor of turbulence induced reduction of terminal velocity due to the motion of the plates may be expressed, for a given spacing of the plates, by an exponential dependence on the mean vibrations speed, one obtains for the particle velocity at the mean hold-up X the following relation

$$u_t = u_0(1 - \bar{X})^{\alpha-1} \exp(-C_1 fA) - U_c/(1 - \bar{X}) . \tag{5}$$

Particle velocity within the plate openings. If the size of the openings in the perforated plate substantially exceeds the particle size one can use Eq. (5) to expres the mean velocity of particles within the plate openings with the following expression for the liquid phase velocity in the plate openings

$$u_p = u_0(1 - \bar{X})^{\alpha-1} \exp(-C_1 fA) - U_c/\varepsilon(1 - \bar{X}) , \tag{6}$$

where ε is specific free area of cross section of the plate.

The pumping effect due to the motion of the plates. If the plates undergo a periodic vibrational motion with an amplitude A, the specific volume of the dispersion pumped across the plate over a single period in either direction amounts to $2A$. As a combined effect of decreased plate free area and longitudinal mixing of the dispersion a hold-up profile develops between the plates. Hence the particle concentration in the space above the plate exceeds that below the plate. On assuming that owing to the intensive mixing of the dispersion in the vicinity of the plate the hold-up above the plate is maintained at a level X_0 and the hold-up below the plate amounts to X_1, the time-averaged flow rate of the solids due to the plate motion may be expressed as

$$U_{pd} = 2\Phi fA(X_0 - X_1) . \tag{7}$$

Since the particle, due to the effect of the inertia and friction forces, does not follow exactly the motion of the liquid, a separating factor Φ has been introduced into Eq. (6). This factor may be expected to be lower than unity but dependent on the particle terminal velocity, u_0.

The flow rate of the solids through the vibrating plate. The expression of the total flow rate of the solids through the plate has been based on the assumption that this rate is a sum of particle flow rates through an immobile plate and that induced by the pumping effect of the plate. The concentration of particles within the plate openings, which affects the former of the two component of the flow rate, varies in the course of the pulsating cycle. Considering that under practical conditions this term is relatively small we shall use for simplicity the mean column hold-up $\bar{X}$ in the following expression

$$U_d = u_p\bar{X} + 2\Phi fA(X_0 - X_1)\,. \tag{8}$$

From this relation it follows that the flow rate of the dispersed phase through the vibrating plate column is strongly coupled with the hold-up profile between the plates. In region far from the flooding the first term in Eq. (8) is usually negligible (may take also negative values at high velocities U_c) and the magnitude of U_d is controlled by the mean hold-up difference $(X_0 - X_1)$.

The distribution of hold-up between the plates. The hold-up distribution between the plates depends on axial mixing of the dispersed phase, induced mainly by the motion of the plates. On characterizing the axial mixing by a dispersion coefficient D, the distribution of the solids between two adjacent plates is described by the following differential equation

$$U_d = u_tX - D(dX/dh) \tag{9}$$

with the following boundary condition

$$h = 0\,, \quad X = X_0\,. \tag{10}$$

The velocity of particles, u_t, as well as the dispersion coefficient D, varies with the distance from the plate, h. The particle velocity varies mainly due to the variable hold-up; the dispersion coefficient due to the decay of turbulence. On assuming that these quantities in Eq. (9) may be approximated by their values corresponding to the mean hold-up $\bar{X}$, this equation can be solved analytically. Thus one obtains a profile of the hold-up as a function of the height above the plate. On expressing the mean hold-up $\bar{X}$ from this profile, the following expressions for the hold-up above and below the plate result

$$X_0 = X_\infty + P\exp(P)(\bar{X} - X_\infty)/[\exp(P) - 1] \tag{11}$$

$$X_1 = X_\infty + P(\bar{X} - X_\infty)/[\exp(P) - 1]\,, \tag{12}$$

where

$$P = u_tH/D \tag{13}$$

$$X_\infty = U_d/u_t\,. \tag{14}$$

From Eqs (*11*) and (*12*) one can express the difference of the hold-ups across the plate by

$$X_0 - X_1 = P(\bar{X} - X_{\infty}) . \qquad (15)$$

In reality though, the effective difference of hold-ups in the expression foi the pumping effect is greater than that given in Eq. (*15*). This is so because part of the particles below the plate, owing to the acceleration gained during the discharge of liquid from the plate, is not returned back into the space above the plate during the reversed motion of the plate. This phenomenon increases the pumping effect in the direction of the solids flow. The excess solids pumping shall be accomodated by a coefficient β and Eq. (*8*) shall be modified to the form

$$U_d = u_p \varepsilon \bar{X} + 2\Phi f A P(\beta \bar{X} - X_{\infty}) . \qquad (16)$$

After expressing P and X_{∞} from Eqs (*13*) and (*14*) one obtains by combining Eqs (*15*) and (*16*) and after some arrangement the following expression

$$U_d/\bar{X} = [\varepsilon u_p + 2\beta\Phi(fAH/D)\, u_t]/[1 + 2\Phi(fAH/D)] . \qquad (17)$$

Upon expressing the dependence of the dispersion coefficient on the speed of vibrations by a power-law expression

$$D \sim (fA)^r \qquad (18)$$

Eq. (*17*) may be modified to the form

$$U_d/\bar{X} = [\varepsilon u_p + \beta C_2(fA)^{1-r}\, u_t]/[1 + C_2(fA)^{1-r}] . \qquad (19)$$

Eq. (*19*) together with Eqs (*5*) and (*6*) express the dependence of the solids flow rate in the vibrating plate column on plate porosity, frequency and amplitude of vibrations, liquid flow rate and the mean hold-up of the solids. In the presented relations there appear three unknown coefficients: β, C_1, C_2 and an exponent r to be determined from experimental data.

EXPERIMENTAL

The Set-up

The measurements were carried out in a 50 mm diameter columm with the effective height of 4 800 mm, equipped with perforated plates. The openings in the plates were circular, 3 mm in diameter with the specific free area 0·05 and 0·1. The scheme of the set-up is shown in Fig. 1. Plates in the column were mounted 1 on a common shaft 2 driven by a vibrator 3 enabling continuous control of the amplitude of the vibrations in the range between 0 and 30 mm and the frequency in the range between 1 and 10 Hz. Water phase was fed at the column bottom from a tank 4 by a pump 5 *via* a heat exchanger 6 and an overflow back into the tank 4. Solid phase, charged through the top end into the column 1 leaves the column through the bottom discharge into an injector 7. Pressurized air was used to drive the solids through pipe 8 into a mechanic separator 9. From here they proceed *via* a vibrating pipe 10 to the column 1, where they are fed below the liquid level. The liquid separated in the separator returns by an auxiliary pipe *via* the heat exchanger 6 back into the injector for the solids transport. The described set-up enables, after setting the flow rate of liquid and after charging the solids, to reach the steady state of the solids flow and the uniform distribution of the hold-up along the column length.

Measured Systems

Two fractions of spherical methacrylate particles and two fractions of glass beads were chosen for measurements. The particles were first screened and subsequently subjected to fluidized bed classification. Terminal velocities of particles were determined by measuring the time for passage of 100 to 200 particles through a one meter long section of the empty column, or, alternatively from measurements of the fluidized bed height under variable liquid flow rate according to Eq. (*1*). Both these methods lead to very similar values which did not differ by more than 2·5%. A very good agreement has been found of the experimental data with Eq. (*1*) which permitted a relatively accurate determination of the exponent for the investigated systems. Characteristic values for the used particles are given in Table I. The liquid phase was in all cases water at the temperature of 25°C.

Procedure

The measurements of the solid hold-up were carried out in the following ranges of variables: amplitude 5—20 mm; frequency 1—10 Hz; hold-up 0·03—0·20; specific plate free area 0·05—0·10; plate spacing 200 mm, for the system IV see Table I also 125 mm; U_d 0·5—2·5 . 10^{-3} m/s; U_c 0—10 . 10^{-3} m/s. The experimental procedure was a follows: After filling the column with water and setting the flow rate of liquid by the metering pump, a definite volume of particles was charged through the opening in the top lid. For given amplitude the required frequency of plate vibrations was set and the feed of air into the injector in the solids circulation loop was turned on. After reaching the steady state (after about 20 minutes) the flow rate of the solids through the

TABLE I
Particle Characteristics

System	Material	Density kg/m^3	Mean diameter 10^{-3} m/s	Terminal velocity 10^{-3} m/s	
I	methacrylate	1 130	0·590	13·70	3·40
II	methacrylate	1 130	0·748	18·04	3·34
III	glass beads	2 670	0·665	97·3	3·05
IV	glass beads	2 670	0·801	120·1	2·62

column was measured in such a way that the outlet stream of the solids from the separator was diverted into a measuring vessel and the volume of discharged solids over a known time interval was recorded. The hold-up of solids in the column was measured in two ways. At higher hold-ups by measuring the height of solids layer settled on the bottom after stopping the flow of phases and plate vibrations. At small hold-ups the solids were discharged after stopping the flow of phases and at lowered plate vibrations into an auxilliary vessel where their volume was measured.

RESULTS

The results of measurements of the solids flow rates in dependence on the mean hold-up and the speed of vibrations are illustrated for selected systems in Fig. 2. A similar dependence was found also for other investigated systems. The experimental data were processed in such a way that the specific flow rate of solids was calculated from Eq. (*19*) while the velocity u_t and u_p were determined from the relations (*5*) and (*6*) for the experimental mean hold-up, $\bar{X}$. The parameters were evaluated by minimizing the sum of square deviations of the experimental and calculated solids flow rate using the Marquardt technique.

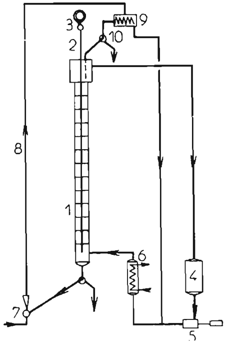

FIG. 1

Scheme of experimental set-up. For description see the Experimental

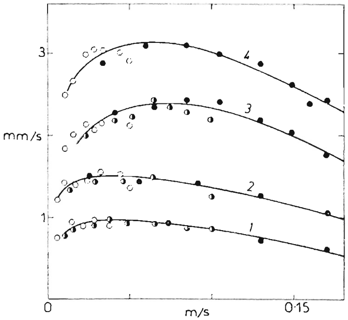

FIG. 2

Specific flow rate of solids, U_d, as a function of the vibration speed fA. $\varepsilon = 0{\cdot}05$, $H = 20$, ○ $A = 5$ mm, ◑ $A = 10$ mm, ● $A = 20$ mm. *1* System II, $\bar{X} = 0{\cdot}05$; *2* system II, $\bar{X} = 0{\cdot}10$; *3* system IV, $\bar{X} = 0{\cdot}05$; *4* system IV, $\bar{X} = 0{\cdot}07$

A preliminary analysis of experimental data from individual series revealed that the data may be well described by Eq. (*19*) with the exponent $r = 0.5$ and the coefficient $\beta = 2.0(1 - X)$. The coefficient C_2 was variable and depended on particle terminal velocity. This dependence could be described by the relation $C_2 \sim u_o^{-1.5}$. Based on these findings Eq. (*19*) could be modified to the form

$$\frac{U_d}{\bar{X}} = \frac{\varepsilon u_p + 2.0(C/u_o)(fA/u_o)^{0.5} u_t}{1 + (C/u_o)(fA/u_o)^{0.5}} . \qquad (20)$$

Eq. (*20*) and Eqs (*5*) and (*6*) describe the dependence of the solids flow rate in the vibrating plate column on the terminal velocity, liquid flow, mean hold-up, plate porosity and the frequency and amplitude of plate vibrations. The presented relations contain two parameters C_1 and C the values of which were determined by processing summarily the data from all series with the result: $C_1 = 2.8 \text{ m}^{-1} \text{ s}$, $C = 0.054 \text{ ms}^{-1}$.

Fig. 3 shows the dependence of the relative flow rates of the solids in the vibrating plate column, related to the solids flow rate in the empty column, on the mean hold-up and the rate of vibrations. The points represent experimental data, solid lines the computed values.

DISCUSSION AND CONCLUSION

The proposed model described the experimental data with a mean error of ±8% comparable with the experimental error of the hold-up and the solids flow rate measurements. Even though the experimental data for the solids flow rate in systems with

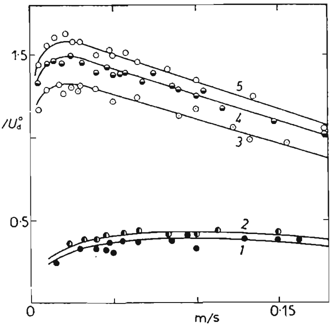

FIG. 3

Solids flow rate scaled by the rate in the empty column at the same mean hold-up as a function of the vibration speed. $\varepsilon = 0.05$, $H = 20$. *1* system III, $\bar{X} = 0.07$; *2* system III, $\bar{X} = 0.03$; *3* system I, $\bar{X} = 0.20$; *4* system I, $\bar{X} = 0.10$; *5* system I, $\bar{X} = 0.05$

high terminal velocities suggest that the frequency and amplitude of the vibrations affect the flow rate in a more complex way than expressed by the product fA, the used simplification is acceptable for the accuracy of the correlation. In view of the number of assumptions and simplifications involved and in view of the limited range of variables the obtained values of parameters cannot be considered as generally valid. Nevertheless, the fact that the values of parameters obtained by processing the experimental data fall into the region of values expected on the basis of physical considerations entailed in the model seems to suggest that the principal factors affecting the column hydrodynamics have been correctly implemented.

As follows from experimental results in Figs 2 and 3 the dependence of the specific flow rate of solids on the speed of vibrations exhibits a maximum for all systems investigated. This maximum is a consequence of the action of competing factors, *i.e.* the increased pumping effect of plates and the decrease of the terminal velocity with increasing intensity of plate vibrations. The effect of solids pumping across the plates may grow to an extent when it increases the flow rate of the solids above a value corresponding to the flow in the empty column at the same mean hold-up (Fig. 3). This effect becomes most conspicuous in region of low particle velocities. It may be expected that similarly as in the case of the solids this pumping effect will also influence the hydrodynamics of liquid dispersions in vibrating plate column and, in particular, the distribution of droplet size near the flooding point when small droplets are being entrained. Under such conditions the large droplets are pumped in the direction of the dispersed phase flow while the backward entrainment of small droplets is increased by the pumping effect in the opposite direction.

LIST OF SYMBOLS

A	amplitude of plate vibrations
C, C_1, C_2	coefficients in Eqs (*20*), (*5*), (*19*)
d	solid particle diameter
D	dispersion coefficient
f	frequency of plate vibrations
h	height above the plate
H	plate spacing
P	Peclet number defined by Eq. (*13*)
r	exponent in Eq. (*18*)
u	particle velocity in empty column
u_0	particle velocity in quiescent liquid
u_p	particle velocity in plate opening
u_r	relative velocity of phases
u_t	particle velocity in vibrating plate column
U_c	superficial velocity of liquid
U_d	specific flow rate of solids
U_{pd}	specific flow rate of solids due to plate motion
X	hold up (volume fraction of solids)

X_0	hold-up above plate ($h = 0$)
X_1	hold-up below plate ($h = H$)
X_∞	limiting hold-up ($h \rightarrow \infty$)
$\bar{X}$	mean hold-up in column
α	exponent in Eq. (*1*)
β	coefficient in Eq. (*16*)
ε	specific plate free area
ϕ	separation factor

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